

The Mathematics of Voting

S.Ananthanarayanan

Elections are round the corner in several states. In most cases, there would be more than one candidate and in many cases, the followers of the one who comes second would say that her votes were ‘split’ with the candidates who ranked even lower. And Pranoy Roy and Co would take opinion polls or exit polls and get into the news anyway!

We follow the Plurality method

In our system, a voter casts just one vote, for one candidate and the winner is the candidate who got the most votes. Except in a case where there are only two contenders, the ‘most votes’ in favour of the winning candidate may not be the majority of the votes. The winner may not even be the most preferred candidate.

Take a case where Ram, Gopal and Lakshmi are standing. Say they get 1000, 900 and 200 votes, respectively. Ram wins, of course, though he got only about 45% of the votes. But suppose the ‘second preference’ of the voters were also considered. Could Gopal, or even Lakshmi, forge ahead?

The Borda count method

In this method voters place the candidates in the order of preference – like Ram, Gopal, Laakshmi, or Ram, Lakshmi, Gopal, or Lakshmi, Ram, Gopal, and so on, A candidate gets 1 point for each last preference, 2 points for each second-last preference, and 3 points for each first preference, in our case of three candidates. The winner is the one with the most points, from all the votes cast.

In our case, it is possible that the 1000 admirers of Ram all had Lakshmi as second preference and the 900 who voted for Gopal also preferred Lakshmi in second place. This would put the scores like this:

Candidate	1 st Preference	2 nd Preference	3 rd Preference	Total points
Ram	1000=3000 pts	200=400 pts	900=900 pts	4200
Gopal	900=2700 pts	200=400 pts	1000=1000 pts	4100
Lakshmi	200=600 pts	1900=3800 pts	0	4400

This time, Lakshmi romps home because she is everybody’s ‘second best’.

Different Constituencies

The problem with the plurality method gets worse when parties field candidates in many constituencies and most constituencies have several candidates. As the party that wins the most seats forms the government, it can happen that the winning party just made it in 50% of the constituencies with only 25% of the votes. And did dismally in the rest of the contests. The party would then rule with only 12.5% of the vote. Another party may have 60% votes in 30% of the seats, or 18% of the vote, but they will lead the opposition!

Mathematics of voting

Donald Saari, a mathematician from Northwestern University, Illinois, has brought the theory of groups, a pretty advanced form of mathematics, to bear on elections. In a two-candidate election, he says, there is 'reflection' symmetry in the distribution of votes for and against. This leads to a clear conclusion. When there are 3 (or more) candidates, then there could still be 'rotational' symmetry, or the contradictions could be put in categories according to where the 'symmetry' collapses.

Arrow and Borda

But there is also the 'Arrow's impossibility theorem' which is a mathematical proof that no election system can satisfy all accepted norms of fairness, once there are more than two candidates. But the Borda method is followed in many clubs and Associations, and even in national elections in Australia.

Borda, incidentally, was a French scientist-surveyor, who pressed for the standard metre and also for uniform measures of weight. He devised the Borda election method for the Academy of Sciences, Paris in 1770.
