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Ramnujan's incredible continued fractions!

S.Ananthanarayanan relates a little incident about Ramanujan, the mathematician, who was in Cambridge during World War I.

At the time, the popular English magazine, *Strand*, carried a regular page of puzzles and problems, under the title of 'Perplexities'. In late August 1914, the Nazis had begun systematic sacking of the Belgian town of Louvain, setting houses on fire street by street and destroying its great library. The *perplexities* column of the December 1914 issue of the *Strand* worked the problems into a story, "Puzzles at a village inn", a report about the German excesses.

A friend's address

The problem was to work out the number of a house on a street of Louvain. The writer spoke of a street where the houses were on one side and were numbered serially, one, two, three, and so on. The writer's friend stayed somewhere on the street, not at its start or end, with houses on both sides. The only thing the writer knew was that all the numbers on one side of the house added up to exactly the same as the numbers on the other side! And *Perplexities* invited readers to work out the friend's house number and visit him in the spring! Another bit of information was that there were at least fifty house in the street, but not as many as five hundred.

It was P C Mahalnobis, who later became the great statistician, who brought the problem to Ramanujan. Mahalnobis himself had used trial and error and had worked it out in a few minutes.

Ramanujan does it his way

Ramanujan got the answer too, at once. But Ramanujan did it by developing one of his celebrated '*continued fractions*', that is, a fraction whose denominator is a number plus a fraction, whose denominator is a number plus a fraction, and so on, ad infinitum!

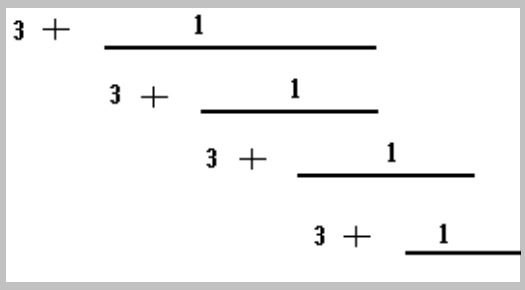
The method immediately gave the only solution to the problem with more than 50 and less than 500 houses – No. 204 in a street of 288 houses. 1+2+3+4+….+203 adds up to 20706, which is the same as 205+206+…+288!

And what is more, Ramanujan's method was the solution at once for the whole class of problems like this. For instance, if the number of houses was 8, then the solution was No. 6, because 1+2+3+4+5 = 15 and 7+8= 15.

Ramnujan's continued fraction

Ramanujan saw that if there were 'n' houses and 'm' was the number of the friend's house, then $1 + 2 + \ldots + (m-1) = (m+1) + (m+2) - \ldots + n$. From this he arrived at a continued fraction in 'n' and 'm', which led him on to the answer!

An example of a continued fraction, also called a ‘staircase’ fraction, would be:



Mahalnobis was thunderstruck. How did you come upon it, he asked. “Oh,” said Ramanujan, “I saw at once that the solution was a continued fraction. “Which fraction?”, I thought, and the answer just came to my mind!”

Ramanujan was a self-tutored genius who did world class work even before he left school but could never clear his first year in university because he would work at nothing but maths! The English mathematician, G H Hardy discovered Ramanujan and took him over to Cambridge for a few, busy years, sadly cut short by Tuberculosis when Ramanujan was just 32 years of age.