appeared on 15th May 2005

Fibonacci numbers in nature

Mathematics and nature seem to be together in making the best use of resources in the world, says S.Ananthanarayanan.

Leonardo Pisano Fibonacci, who lived in Italy in the 12th and 13th centuries, discovered a series of numbers, just numbers increasing by a simple formula, which still displayed remarkable relationships. It is found that this series of numbers also appears in many parts of the natural world, and with good reason!

Fibonacci numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233……………..

In this series, each number is the sum of the previous two numbers. We can see that 2=1+1, 3=2+1, 5=3+2, 8=5+3…. 233=144+89 and so on.

One of the things about the series is that each number depends on its predecessor, which depends on its predecessor, which depends on its predecessor, and so on, a *regressive* relationship. But a more practical property of the series is that the ***ratio*** of successive terms tends to become the same, as we go higher into the series.

The ratios

The ratios of the successive pairs in the series are:

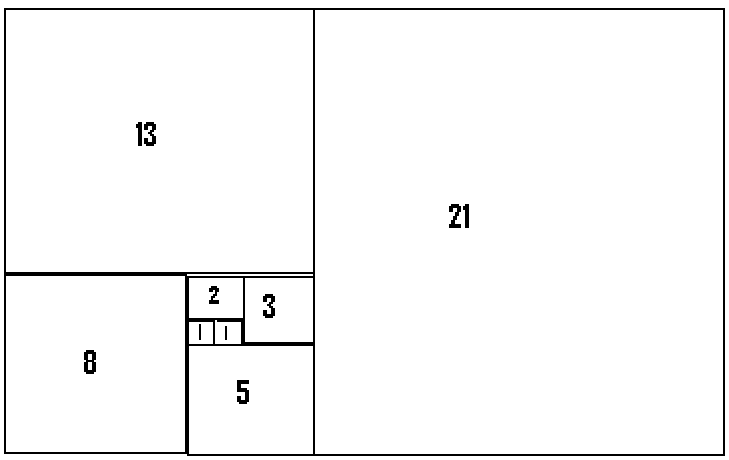
1/1=1	2/1=2	3/2=1.5	5/3=1.666
8/5=1.60	13/8=1.625	21/13=1.615	34/21=1.619
55/34=1.6176	89/55=1.6182	144/89=1.61797	233/144=1.61806

We can see that the ratio is getting closer and closer to **1·61803 39887**..., which is the value we reach when we go very deep into the series and work out the ratio. It can be proved that the exact ratio can never, even in theory, be exactly expressed. This is to say that the numbers after the decimal place never end, and they do not even repeat, like we have with numbers like 2/3=0.66666…. ,or 9/7=1.28571428…., where the 6 digits after the decimal keep repeating.

A ratio like this, which is not the ratio of any two whole numbers, is called an irrational number. The ratio of the Fibonacci numbers is called the golden section or the golden ratio or the golden mean or even the divine proportion.

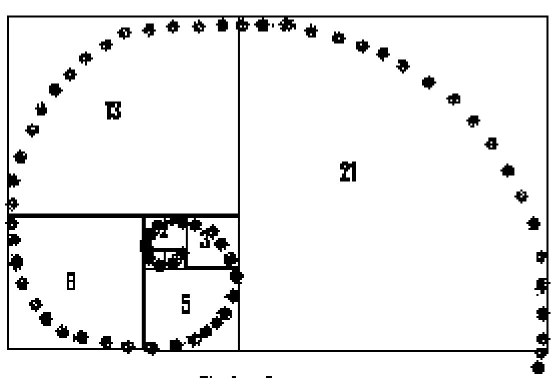
Fibonacci in nature

One way of representing Fibonacci numbers is with squares. In the picture, the side of the ‘2’ square is made by putting the ‘1’ squares togethe . The side of the ‘3 square is then made from the ‘2’ square and the ‘1’ square, the ‘5’ square from the ‘3’ and ‘2’ squares, the ‘8’ square from the ‘5’ square and the ‘3’ square, and so on.



Squares

Yet another way is draw a spiral, by drawing a quarter circle in each square, followed by a larger quarter circle in the next square, and so on, like in the next picture.



Spiral

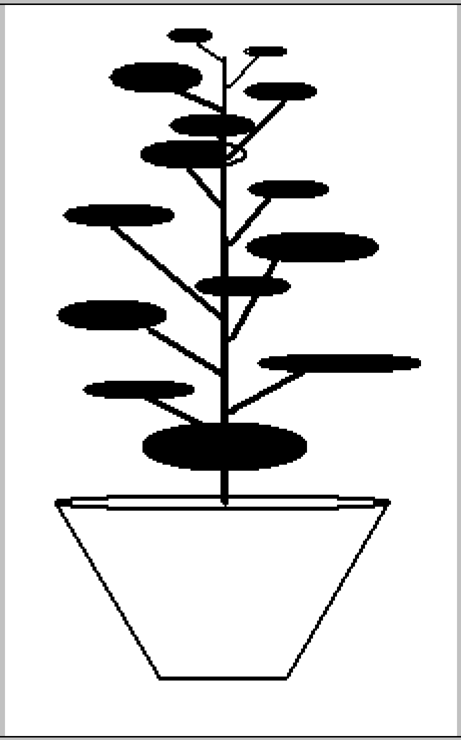
Nautilus sea shell

This is very close to spirals often seen in nature, like in snail shells, sea shells and even the arrangement of seeds and the leaves and branches of plants. The form of natural things seem to favour a ratio of 1.618…. in the size of successive turns in the shape, just like the sizes of the squares, which are the Fibonacci numbers. But why this particular ratio, one in fact that cannot be expressed as a fraction of numbers?

Arrangement of leaves

The reason seems to be that Fibonacci numbers help the components of natural systems keep their identity even as the system increases in dimensions.

The leaves of a plant, as one goes up from the lower leaves to the higher, are arranged not one below the other, but each a little to the side. It seems to be way to help each leaf catch as much sunlight as possible and to make each leaf most useful in channeling rainwater to the roots. It is found that the extent of the turn, as one goes from one leaf to the next are Fibonacci numbers. The purpose seems to be to make the ratios different from whole number fractions, that is, an irrational number.



In this way, one leaf will not come directly over another, even after many tiers of growth, and the plant makes the best use of sunlight and. rainfall

1/1=1	2/1=2	3/2=1.5
5/3=1.666	8/5=1.60	13/8=1.625
21/13=1.615	34/21=1.619	55/34=1.6176
89/55=1.6182	144/89=1.61797	233/144=1.61806