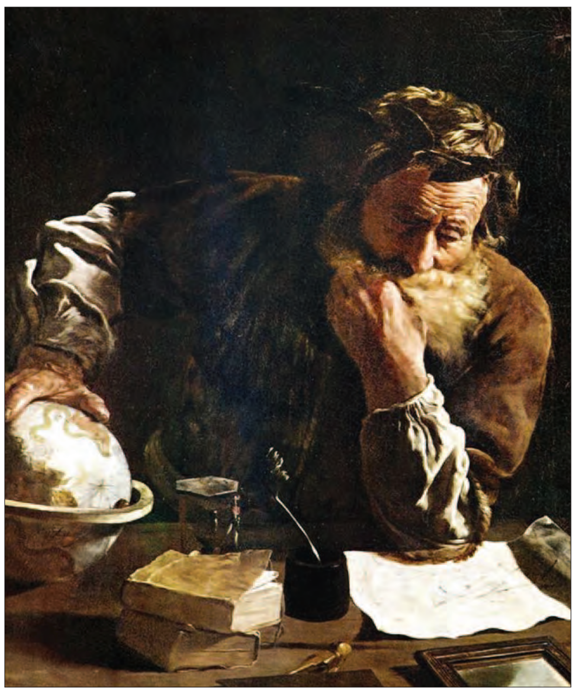
The day of pi

THERE IS HIGHER MATH HIDDEN BEHIND THE SIMPLE RATIO OF THE CIRCLE TO ITS RADIUS, WRITES **S ANANTHANARAYANAN**

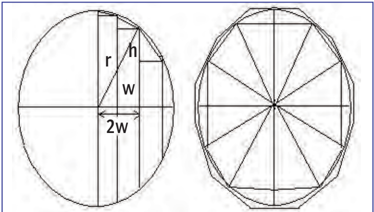
A special day to celebrate *pi*, the number denoted by the symbol π and known to most as 22/7 or the value 3.14 in decimals, was observed on 14 March. This was in the USA and such places where the date is written month-first, as 3/14, which reads out the usual value of pi. And 14 March 2015 was even more special because the date read 3/14/15, which gives the value of pi to four decimal places and happens only once in a century. Even more memorable was the date, 14 March 1592, four centuries ago, because 3.141592 is the value of pi to six decimal places! But even in our own century, "mass elation will peak at 9.26:53 am when the date and time will describe pi to 10 digits," said *The Guardian* newspaper. "This only happens once in 100 years, so it is likely your only chance to celebrate," said the "learning" part of the *Bangkok Post* website.

But apart from being a mathematical value that fits into the calendar in this way, the number pi has features that make it quite a star, even otherwise, among mathematical things. The purpose of naming a day after pi and celebrating the day by eating pies, for instance, one hopes, is to use fun and food to draw attention to different qualities of this important number.

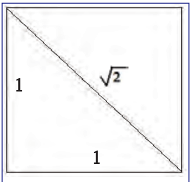
The first property of the number, of course, is that it is the ratio of the diameter of a circle to the circumference ($c=2\pi r$). The fact that it is also the ratio of the square of the radius to the area of the circle ($area=\pi r^2$) would suggest how this ratio arises. But at first it was worked out simply as the constant ratio that masons and carpenters had noticed in the course of their work, and this value is approximate, but practical. It is perhaps Archimedes who first used a mathematical method to work out the ratio without recourse to the actual measurement of circles. This way of working things out is called the *analytical* method.


This 1620 painting by Domineco Fetti catches a thoughtful Archimedes at work.

One direct way of doing this is to divide the circle into strips, like in the picture. Now, if each strip is taken as having some width, then the distance of the strip from the centre is known and, hence, using principles of geometry its height is also known. With the help of the height and the width, we can work out the area of each strip. Then, adding together the area of all the strips leads us somewhat near the area of the circle. Now if we make the strips narrower and narrower, we can work out the area of the circle more accurately, in terms of the radius.



Using 'stripes' Archimedes' method



A line of length $\sqrt{2}$ can be drawn, but the exact value cannot be worked out

We can similarly, work out very nearly the circumference of the circle by adding up all the slanting ends of the strips, as the strips get narrower and narrower. Archimedes actually did something like this, only he worked out the area, or the sides, of a polygon, which is a many-sided shape, fitted inside and outside the circle, and then increased the number of sides to make the shape more nearly like the circle.

In later centuries, with the advance in mathematics, the principles of these methods have been refined and we have exact formulas. The *Liebniz* (1646-1716) formula, for instance, gives the value of pi as π/4=1-1/3+1/51/7+1/9—1/11+1/13- π to infinite terms. A feature we can notice of this series is that every odd, and positive, term is slightly greater than the adjacent, negative term. The value of the series hence gradually increases to approach an exact value of pi, the addition by the terms further down in the series being smaller and smaller.

But the trouble with this formula is that the approach to the correct value is slow and it takes many, many terms before the value is good enough. Better formulas have been developed by different ways of working out the areas in the elementary strips or polygons, and these are now used for the most accurate computation of the value of pi.

One method was even "experimental" — of dropping a needle on to a grid of parallel lines and counting the number of times the pin fell over a line. It has been worked out that if the lines are drawn one unit apart and the length of the pin, k, is less than this unit, then the probability that the pin will cross a line is $2k/\pi$. Throwing the pin, which can be automated, a huge number of times can thus generate an accurate value for π.

Is the value important?

The value, beyond a few decimal places, is of no practical value, except in large surveys or astronomy where more decimal places are needed. But there is great academic interest in the fact that the exact value of π can never ever be evaluated. This is because the correct value is an infinite series of non-repeating digits after the decimal point, never evaluating to an exact value. Such a quality of a number amounts to saying that it can never be correctly expressed as a ratio of two integers, which are not multiples of each other, like 7/3, or 13/5 or even 22/7. All fractions like this, where one whole number divides another, can be evaluated as an exact decimal number, or as a recurring decimal, where the division keeps giving the same remainder or the same series of remainders. Examples of recurring decimals are 1/3=0.3333333, or 31/99=0.31313131…

Numbers like this, which cannot be expressed as a ratio, are called *irrational numbers*. There are more examples, a ready one being the square root of the number 2. This example is easily proved, as this square root could be expressed as a ratio, then the squares of the two integers in the ratio would divide into the number 2. And there is no such pair of squares.

Random progression

An important application of this kind of number, whose decimal expansion is infinite, is that the progression of digits in the decimal form is essentially random. This must be so, as else there would be a endless repetition of patterns. Irrational numbers are thus a source of randomness and a computer generated value, running into millions of digits, could be used as a code that would be very difficult to break unless the eavesdropper knew at which stage of the progression the code-maker had started.

These properties of pi and other irrational numbers are the same, even if we change the system of counting from the decimal, or on the base of 10, to another like the binary, based on 2, or the Octal, based on 8, or the hexadecimal, based on 16, which computers use. The properties, in fact, represent basic features of circles, lines and angles and even geometries in more than three dimensions and the study of pi is of great and fundamental importance.

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The value therein

A mnemonic for pi to 11 digits goes like this, "Now I know a rhyme excelling, of mystic words and magic spelling." The number of letters in the words of this mnemonic spell them out:
3.14159265358.
Computers, of course, can do much better:
Pi=3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164 0628620899 8628034825 3421170679 (pi to 100 decimal places).

Or:
3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164 0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 5058223172 5359408128 4811174502 8410270193 8521105559 6446229489 5493038196 (pi to 200 decimal places).

CELLULAR INFORMATION

TAPAN KUMAR MAITRA EXPLAINS THE STRUCTURAL BASIS OF DNA, CHROMOSOMES AND THE NUCLEUS

Cellular structure and function involve a sense of predictability, order and control. We have come to expect that organelles and other cellular structures will have a predictable appearance and function, that metabolic pathways will proceed in an orderly fashion in specific intracellular locations and that all of a cell's activities will be carried out in a carefully controlled, highly efficient and heritable manner.

Such expectations express our confidence that cells possess a set of "instructions" that specify their structure, dictate their functions and regulate their activities — and that these instructions can be passed on faithfully to daughter cells. More than 100 years ago, Augustinian monk Gregor Mendel worked out rules accounting for the inheritance patterns he observed in pea plants, although he had little inkling of the cellular or molecular basis for these rules. These studies led him to conclude that hereditary information was transmitted in the form of distinct units that we now call genes. We also now know that genes consist of DNA sequences that code for functional products that are usually protein chains, but may in some cases be RNA molecules that do not code for proteins.

The information carried by DNA flows both between generations of cells and within each individual cell. During the first of these two processes, the information stored in a cell's DNA molecules undergoes replication, generating two DNA copies that are distributed to the daughter cells when

the cell divides. Instructions stored in DNA are utilised in a two-stage process called transcription and translation. During transcription, RNA is synthesised in an enzymatic reaction that copies information from DNA. During translation, the base sequences of the resulting messenger RNA molecules are used to determine the amino acid sequences of proteins. Thus, the information initially stored in DNA base sequences is ultimately used to code for the synthesis of specific protein molecules.

It is the particular proteins synthesised by a cell that ultimately determine most of its structural features as well as the functions it performs. The discovery of DNA dates back to the early studies of Friedrich Miescher, but it wasn't until the mid-20th century that experiments with pneumococcal bacteria and bacteriophage T2 clearly revealed DNA to be the genetic material. This work was followed by James Watson and Francis Crick's elucidation of the double-helical structure of DNA, one of the landmarks in 20th century biology. The hydrogen bonds holding together the two strands of the double helix only fit when the base A is paired with T, and the base G is paired with C. Because the two are held together by relatively weak, noncovalent bonds, they can be readily separated during DNA replication and RNA synthesis.

Molecular biologists have developed a number of powerful tools for studying DNA. For example, restriction enzymes isolated from bacteria can be used to cut DNA into reproducible fragments that are short enough to be easily manipulated in the laboratory. The DNA (or RNA for some viruses) that makes up one complete set of an organism's genetic information is called its genome. For most viruses and prokaryotes, the genome consists of a single DNA molecule or a small number of them. Eukaryotes have a nuclear genome divided among multiple chromosomes, each possessing one very long DNA molecule. Eukaryotes also have a mitochondrial genome and, in the case of plants, a chloroplast genome.

Restriction enzymes isolated from bacteria can be used to cut DNA into reproducible fragments that are short enough to be easily manipulated in the laboratory. And automated DNA sequencing techniques have allowed scientists to determine the entire base sequence of the genomes of numerous organisms, from bacteria to humans. One of the most striking features of the nuclear genomes of eukaryotes, especially multicellular organisms, is the large fraction of DNA that does not code for RNA or protein synthesis. Much of this noncoding DNA consists of repeated sequences. Relatively little is known about the functions of repeated DNA, but some of these sequences may play a structural role in the chromosome, and others may provide sources of evolutionary variability for the genome.

The enormous length of the DNA molecules present in cells (and even viruses) necessitates considerable packaging. In both prokaryotes and the nuclei of eukaryotic cells, the DNA is complexed with proteins, but this packaging is more elaborate in eukaryotes. The basic structural unit of the eukaryotic chromosome is the nucleosome, which consists of a short length of DNA wrapped around a protein particle constructed from eight histone molecules. Stretches of nucleosomes ("beads on a string") are packed together to form a 30-nm chromatin fiber, which can then loop and fold further. The more highly compacted the DNA, the less likely it is to be transcriptionally active in the cell.

In non-dividing cells that are actively transcribing DNA, much of the chromatin is in a relatively extended, highly diffuse form called euchromatin. However, other portions of chromatin are in a highly condensed state called heterochromatin. During cell division, all the chromatin becomes highly compacted, forming discrete chromosomes visible with the light microscope.

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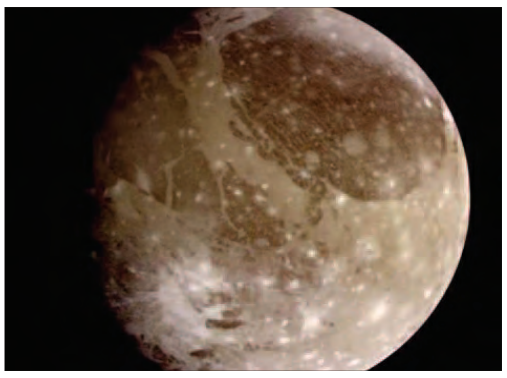
Ganymede and alien life

USING PIONEERING TECHNIQUES, NASA SCIENTISTS HAVE FOUND WATER BENEATH THE MOON'S CRUST OF ICE, SAYS **ANDREW GRIFFIN**

National Aeronautics and Space Administration scientists have confirmed that Jupiter's moon, Ganymede, has an ocean beneath its icy crust and could have harboured life. While the fraternity has long suspected there was water beneath the surface — ever since the *Galileo* spacecraft provided hints during its exploration of earth's satellite and its moons from 1995 to 2003 — Nasa scientists have confirmed the theory using new data.


This natural color view of Ganymede was taken from the Galileo spacecraft during its first encounter with the Jovian moon.

"It is one step further toward finding that habitable, water-rich environment in our solar system," said astronomer Heidi Hammel with the Washington-based Association of Universities for Research in Astronomy. It is the latest indication that many of the objects in our solar system have water and could support life — earlier last week, scientists confirmed that Saturn's moon had hot springs underneath its crust.

To make their discovery, the scientists had to work backwards from the visuals seen on the moon. Like earth, Ganymede has a liquid iron core that generates a magnetic field, though this field is embedded within Jupiter's magnetic field. That sets up an interesting dynamic with telltale visuals — twin bands of glowing aurora around Ganymede's northern and southern polar regions.

As Jupiter rotates, its magnetic field shifts, causing Ganymede's aurora to rock. Scientists measured the motion and found it fell short. Using computer models, they realised that a salty, electrically conductive ocean beneath the moon's surface was counteracting

Jupiter's magnetic pull. They used more than 100 computer models to understand whether the aurora could be affected by anything else, and repeated many of the observations. After that, they were confident that what they had seen indicated the sub-ice water.

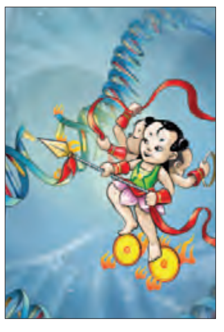
"They developed a new approach to look inside a planetary body with a telescope," said Nasa planetary science division director Jim Green.

THE INDEPENDENT

PLUS POINTS

Red hot

Scientists have a fever for genome tinkering, and the latest thing shooting up temperatures is Clustered, Regularly Interspaced, Short Palindromic Repeats. The number of publications referring to CRISPR/Cas technology has mushroomed since its first use as a genome-editing tool in 2012. In a testament to the method's popularity, a recent guest lecture at Vanderbilt University by CRISPR pioneer and 2015 Breakthrough Prize-winner Jennifer Doudna packed a 300-person classroom and a 160-person overflow room — which then itself overflowed, recalls attendee Douglas Mortlock, a research assistant professor at Vanderbilt who blogs about advances in CRISPR technology.



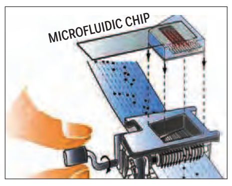
Named for particular DNA loci that are found in many archaea and bacteria, CRISPR works with associated nucleases, including Cas9, to protect the cells from viral infection by inserting short snippets of viral DNA into the CRISPR cassette. By combining the Cas9 nuclease with a short guide RNA that's custom-designed to bind a specific target, CRISPR/Cas can easily edit any gene you want. Just in the past year, for example, it has allowed researchers to cure a rare liver disease in mice, to excise HIV-inserted genes from human immune cells, and to block HIV from entering blood stem cells.

CRISPR/Cas is easier than the other nuclease-based editing technologies, says John Schimenti of Cornell University.

KELLY RAE CHI/THE SCIENTIST

Within reach

Researchers at Stanford University have designed a new microfluidic device that is powered by hand crank and programmed by paper punch cards, which they described in PLOS One on 4 March. Riffs on the device, based on a design for a children's music box, should allow researchers to create portable, sturdy microfluidic devices for use in the classroom, as well and for diagnostic testing and environmental monitoring in areas with limited access to electricity and trained personnel.



"This paper is the most clever use of an inexpensive programmable toy to produce useful microfluidic systems at low cost," Paul Yager, a professor of bioengineering at the University of Washington in Seattle, who was not involved in the study, wrote in an e-mail to *The Scientist*. "This has a lot of potential for teaching and for performing processes at low cost in the developing world."

Manu Prakash, co-author of the new paper and an assistant professor of bioengineering at Stanford, said that his team's device managed to have high-tech capabilities despite its relatively low-tech operational requirements. "The tool has exactly the same specs as any of these other microfluidic technologies that are out there that are all automated and run by computers," he said.

KATE YANDELL

Natural quasicrystal

A team from Princeton University and the University of Florence in Italy has discovered a quasicrystal — so named because of its unorthodox arrangement of atoms — in a 4.5-billion-year-old meteorite from a remote region of northeastern Russia, bringing to two the number of natural quasicrystals ever discovered. Prior to the team finding the first natural quasicrystal in 2009, researchers thought that the structures were too fragile and energetically unstable to be formed by natural processes.


The first naturally occurring quasicrystal exhibited a fivefold symmetry, where each point has a fivefold axis of symmetry.


The recently discovered quasicrystal has a tenfold symmetry and resembles stacked 10-sided structures.

"The finding of a second naturally occurring quasicrystal confirms that these materials can form in nature and are stable over cosmic time scales," said Paul Steinhardt, Princeton's Albert Einstein Professor of Science and a professor of physics, who led the study with Luca Bindi of the University of Florence. The team published the finding in the 13 March issue of the journal *Scientific Reports*.